Automatic Control (2)

By

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Electrical Engineering and Control Program

كلية الهندسة بشبرا

FACULTY OF ENGINEERING AT SHOUBRA

Lecture (1)

By

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Course Title: Automatic Control (2)

Course Code: EEC 415

Prerequisites: EEC224 Signals and Systems

Study Hours: 3 Cr. hrs.

= [2 Lect. + 0 Tut + 3 Lab]

Assessment:

Final Exam: 40%.

Midterm: 30%.

Quizzes: 10%.

Home assignments and Reports: 10%.

MATLAB Mini Project: 10%.

Textbook:

1- K. Ogata, Modern Control Engineering, Pearson, 5th. Ed., 2009.
2- Nise, N. S. “Control System Engineering”, 7th edition, John Wiley & Sons Ltd., UK, 2016.
3- F. Golnaraghi and B. C. Kuo, “Automatic control Systems”, 10th ed., John Wiley & Sons, Inc. 2017.
4- Andrea Bacciotti, “Stability and Control of Linear Systems” Volume 185, Springer, 2019.

Course Description

- Compensation in control systems, lead, lag, and lead-lag phase compensation in frequency domain,
- State model of linear systems using physical variables, state space representation using phase variables, state space representation, using canonical variables, properties of transition matrix and solution of state equation,
- Poles, zeros, eigen values and stability in multivariable system,
- Introduction to nonlinear control systems, describing function method, nature and stability of limit cycle.

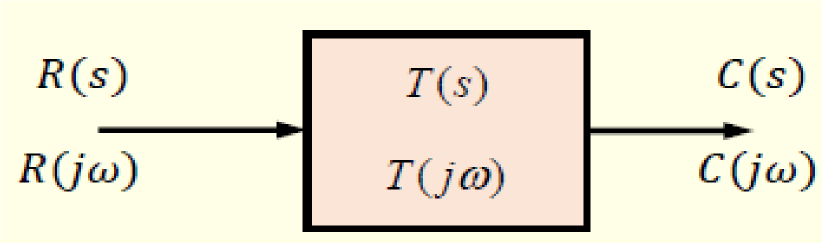
Analysis & Design of Control Systems using Frequency Response

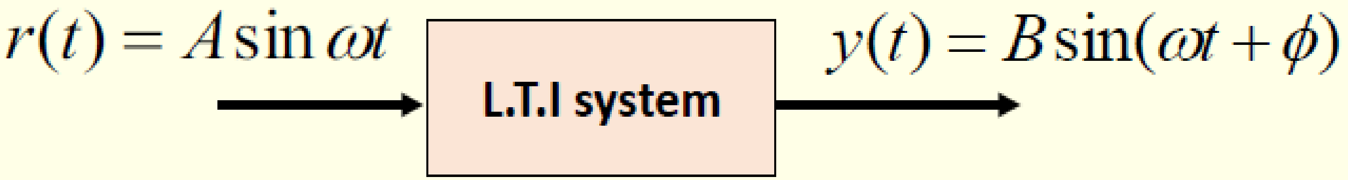
Frequency Response

- For a stable, linear, time-invariant (LTI) system, the steady state response to a sinusoidal input is a sinusoid of the same frequency but possibly different magnitude and different phase.

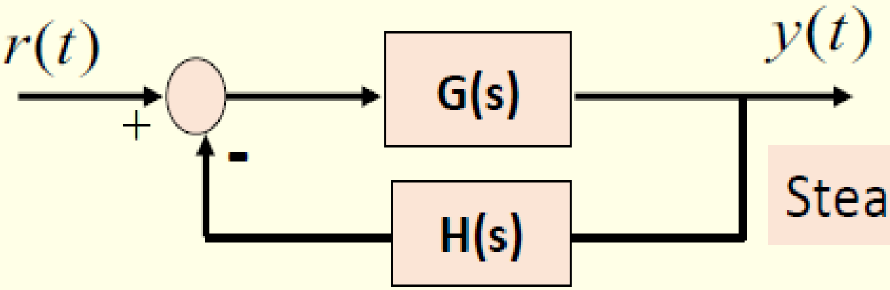
- Sinusoids are the Eigen functions of convolution.

- If input is $A\ cos(\omega_o t + \theta)$ and steady-state output is B cos($\omega_o t$ + φ), then the complex number $\frac{B}{A}e^{j(\varphi-\theta)}$ is called the frequency response of the system at frequency ω_o.





Magnitude: $\frac{B}{A}$ **Phase:** ϕ



Steady state response

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$s = \sigma + j\omega \Rightarrow s = j\omega$$

Magnitude: $\frac{|G(j\omega)|}{|1+G(j\omega)H(j\omega)|}$ **Phase:** $\frac{\angle G(j\omega)}{\angle[1+G(j\omega)H(j\omega)]}$

Bode Plots

- ***The Bode form*** is a method for the frequency domain analysis.

- ***The Bode plot*** of the function G(jω) is composed Bode of two plots:

➢ One with the magnitude of G(jω) plotted in decibels (dB) versus $\log_{10}$ (jω).

➢ The other with the phase of G(jω) plotted in degree versus $\log_{10}(\omega)$.

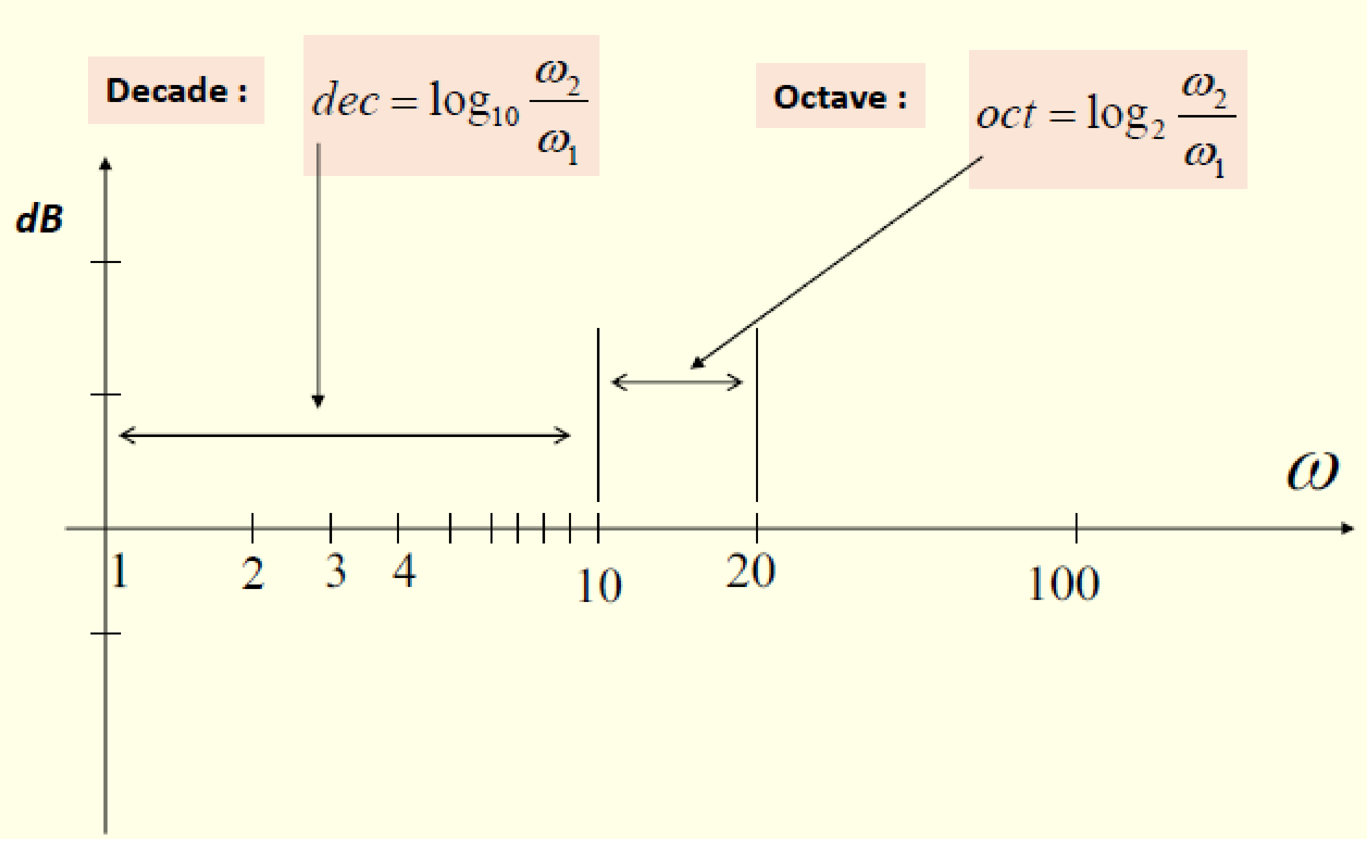
Feature of the Bode Plots

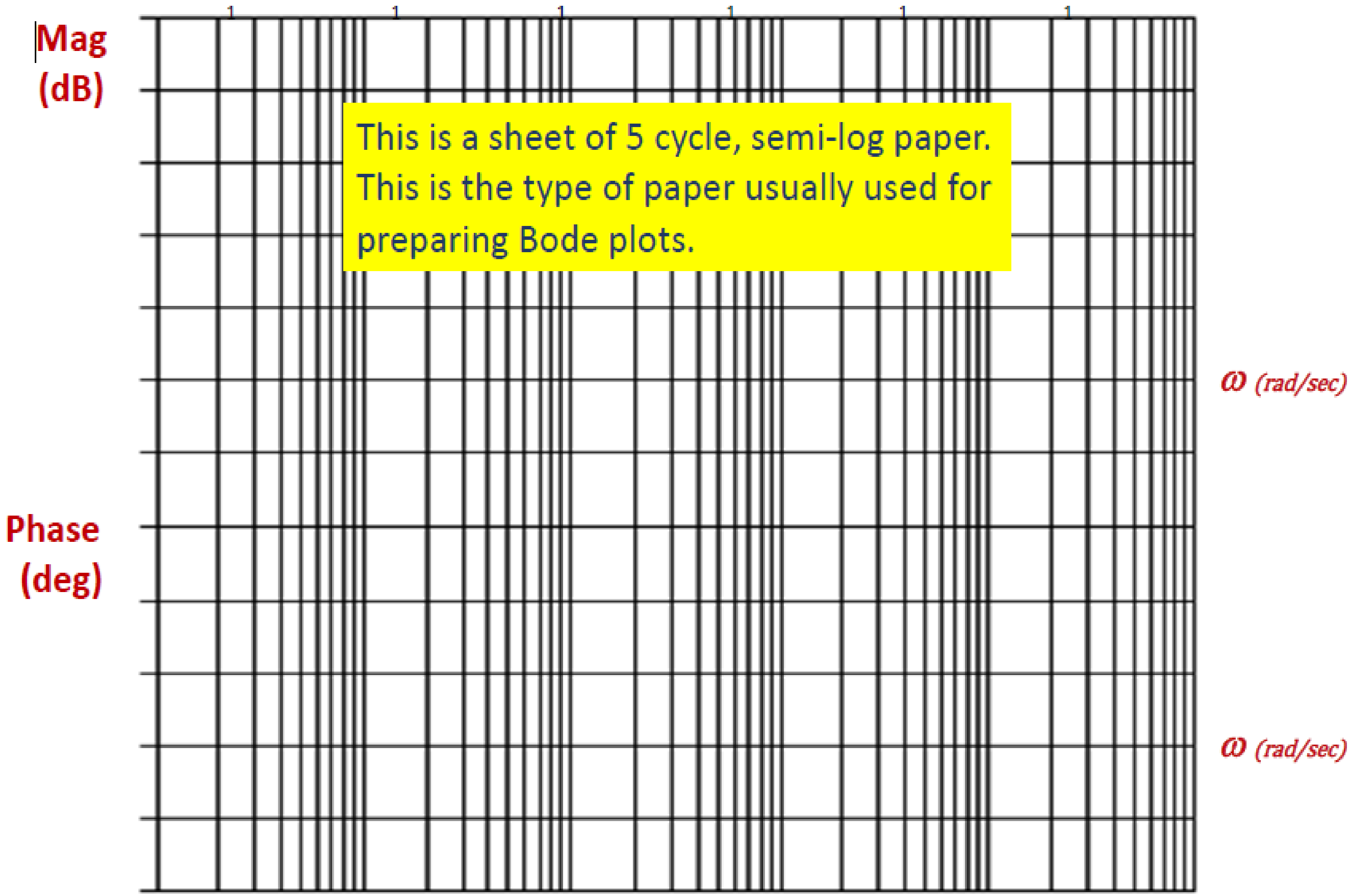
- Since the magnitude of G(jw) in the Bode plot is expressed in dB, products and division factors in G() become ***additions and subtraction, respectively.***

- The phase relations are also added and subtracted from each other algebraically.

- The magnitude plot of Bode of G(jw) can be approximated by ***straight lines segments*** which allow the simple sketching of the bode plot without detailed computation.

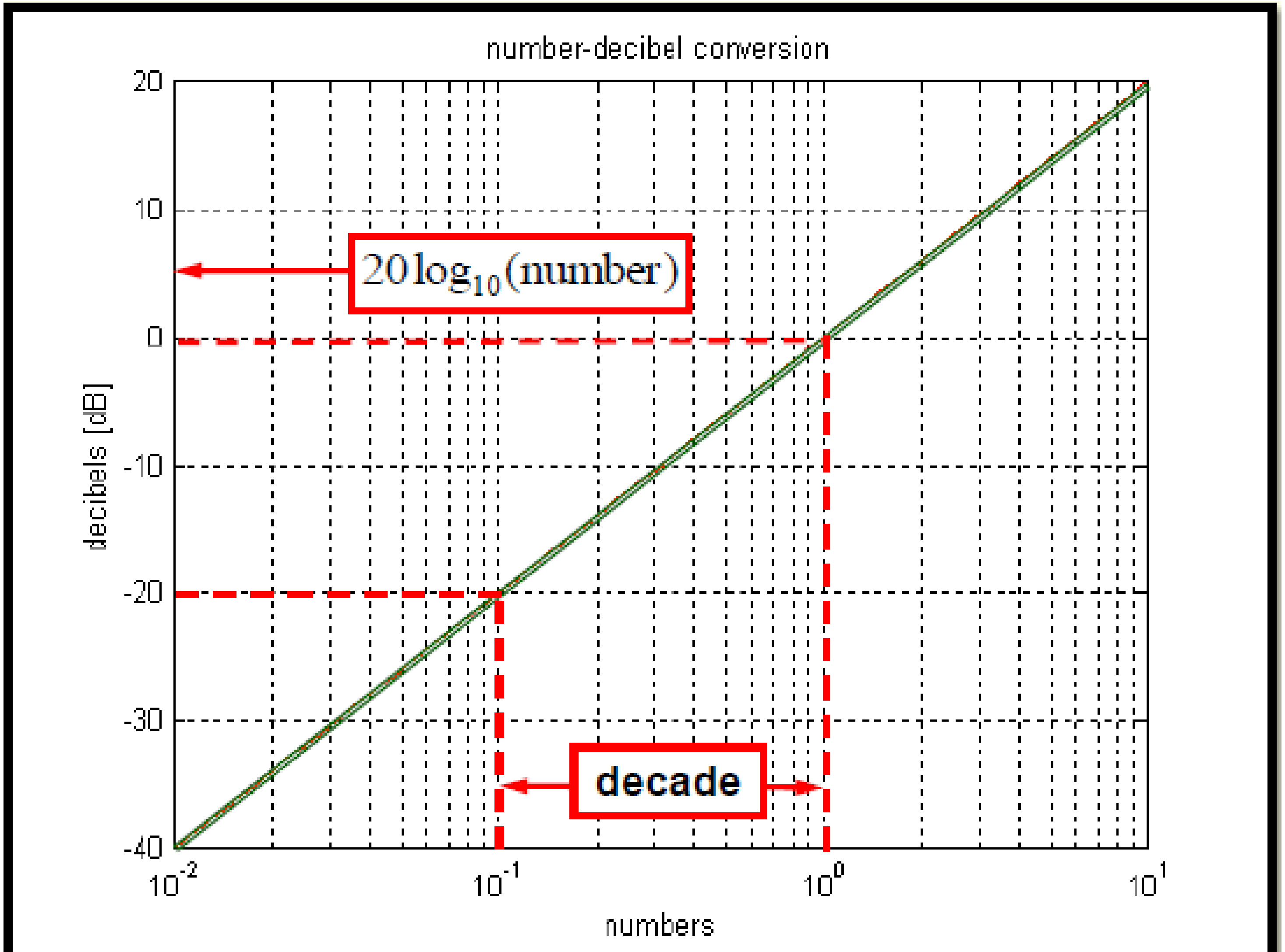
Logarithmic coordinate





This is a sheet of 5 cycle, semi-log paper. This is the type of paper usually used for preparing Bode plots.

A number to decibel conversion



Bode Plots

- In order to simplify the Bode plot, it is convenient to use the so-called ***Bode form***.

- **Given**: the open loop T.F

$$G(s)H(s) = \frac{A(s+z_1)(s+z_2)\ldots(s+z_m)}{s^q(s+p_1)(s+p_2)\ldots(s+p_n)(s^2+as+\omega_o{}^2)}$$

Where A, m, q and n are real constants.

- The bode form can be expressed as:

$$G(s)H(s) = \frac{K\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\ldots\left(1+\frac{s}{z_m}\right)}{s^q\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\ldots\left(1+\frac{s}{p_n}\right)\left[1+\frac{a}{\omega_o}s+\left(\frac{s}{\omega_o}\right)^2\right]}$$

$$G(j\omega)H(j\omega) = \frac{K\left(1+\frac{j\omega}{z_1}\right)\left(1+\frac{j\omega}{z_2}\right)\ldots\left(1+\frac{j\omega}{z_m}\right)}{j\omega^q\left(1+\frac{j\omega}{p_1}\right)\left(1+\frac{j\omega}{p_2}\right)\ldots\left(1+\frac{j\omega}{p_n}\right)\left[1+\frac{a}{\omega_o}j\omega+\left(\frac{j\omega}{\omega_o}\right)^2\right]}$$

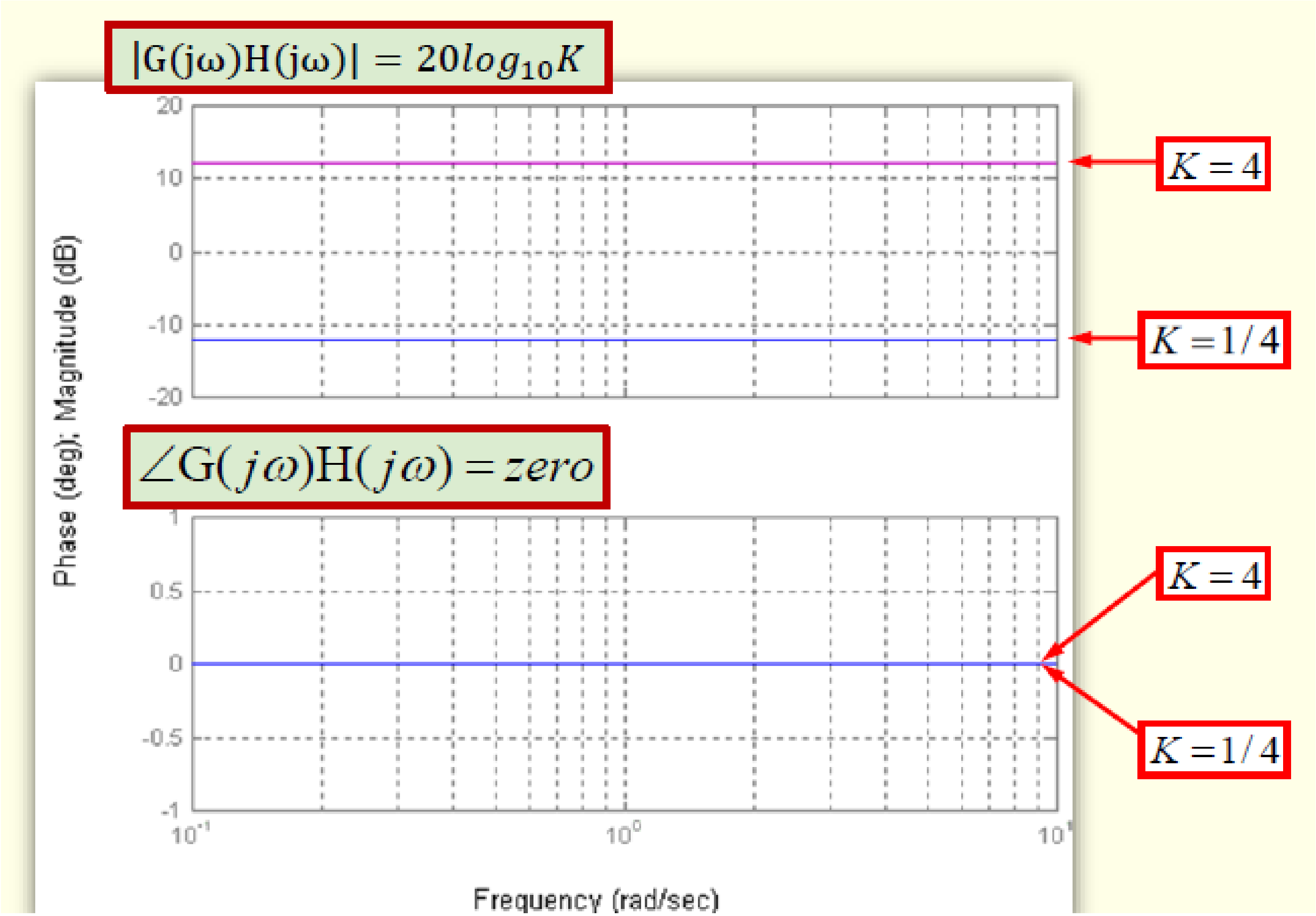
Basic Terms

- Constant gain $G(s)H(s) = K$ or $G(j\omega)H(j\omega) = K$
- Integral factors $G(s)H(s) = \frac{1}{s}$ or $G(j\omega)H(j\omega) = \frac{1}{j\omega}$
- Derivative factors $G(s)H(s) = s$ or $G(j\omega)H(j\omega) = j\omega$
- First order factors $G(s)H(s) = \left(1 + \frac{s}{a}\right)^{\pm 1}$ or $G(j\omega)H(j\omega) = \left(1 + \frac{j\omega}{a}\right)^{\pm 1}$
- Quadratic factors $G(s)H(s) = \left[1 + \frac{2\zeta}{w_n} s + \left(\frac{s}{w_n}\right)^2\right]^{\pm 1}$

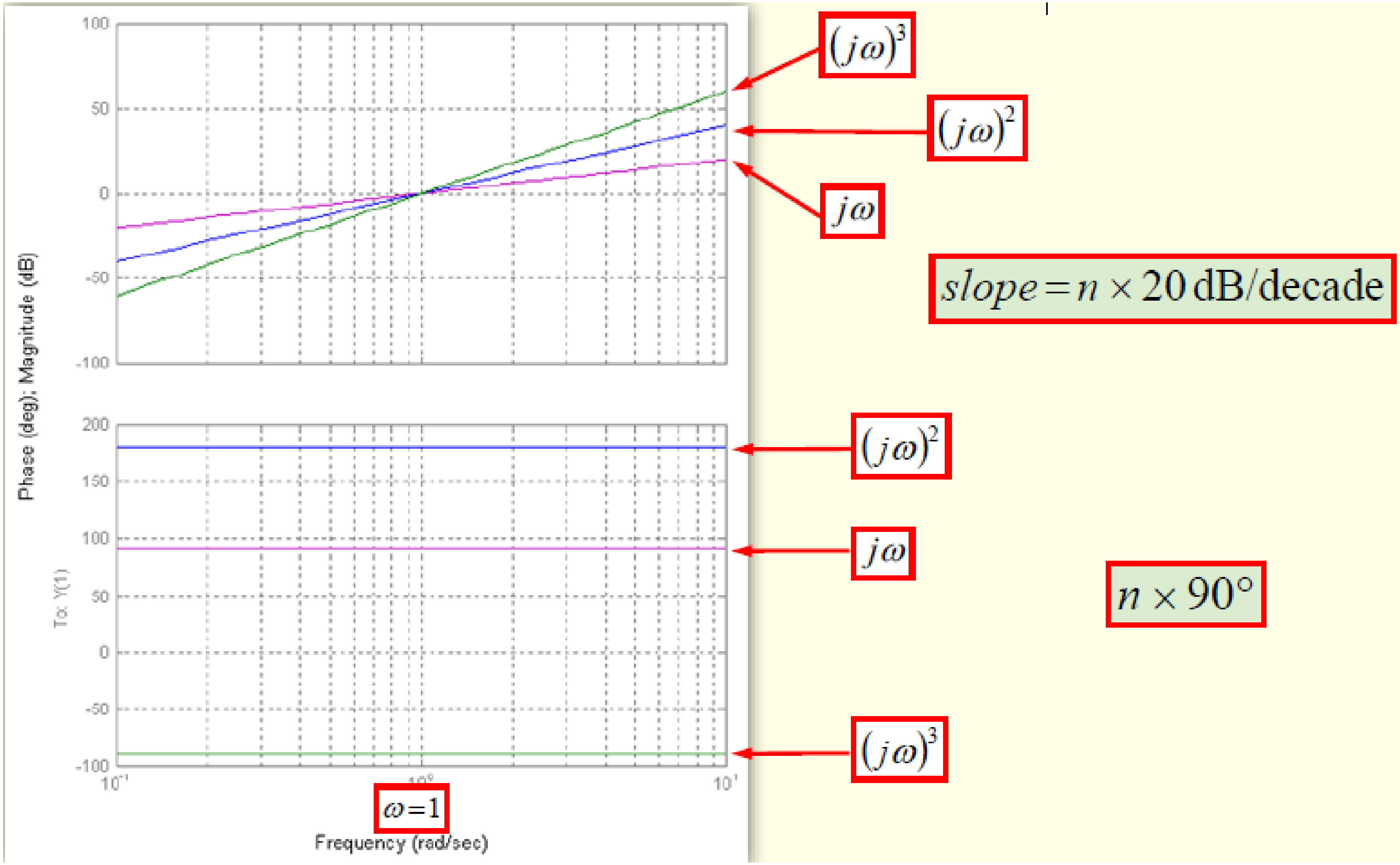
or

$$G(j\omega)H(j\omega) = \left[1 + \frac{2\zeta}{w_n} j\omega + \left(\frac{j\omega}{w_n}\right)^2\right]^{\pm 1}$$

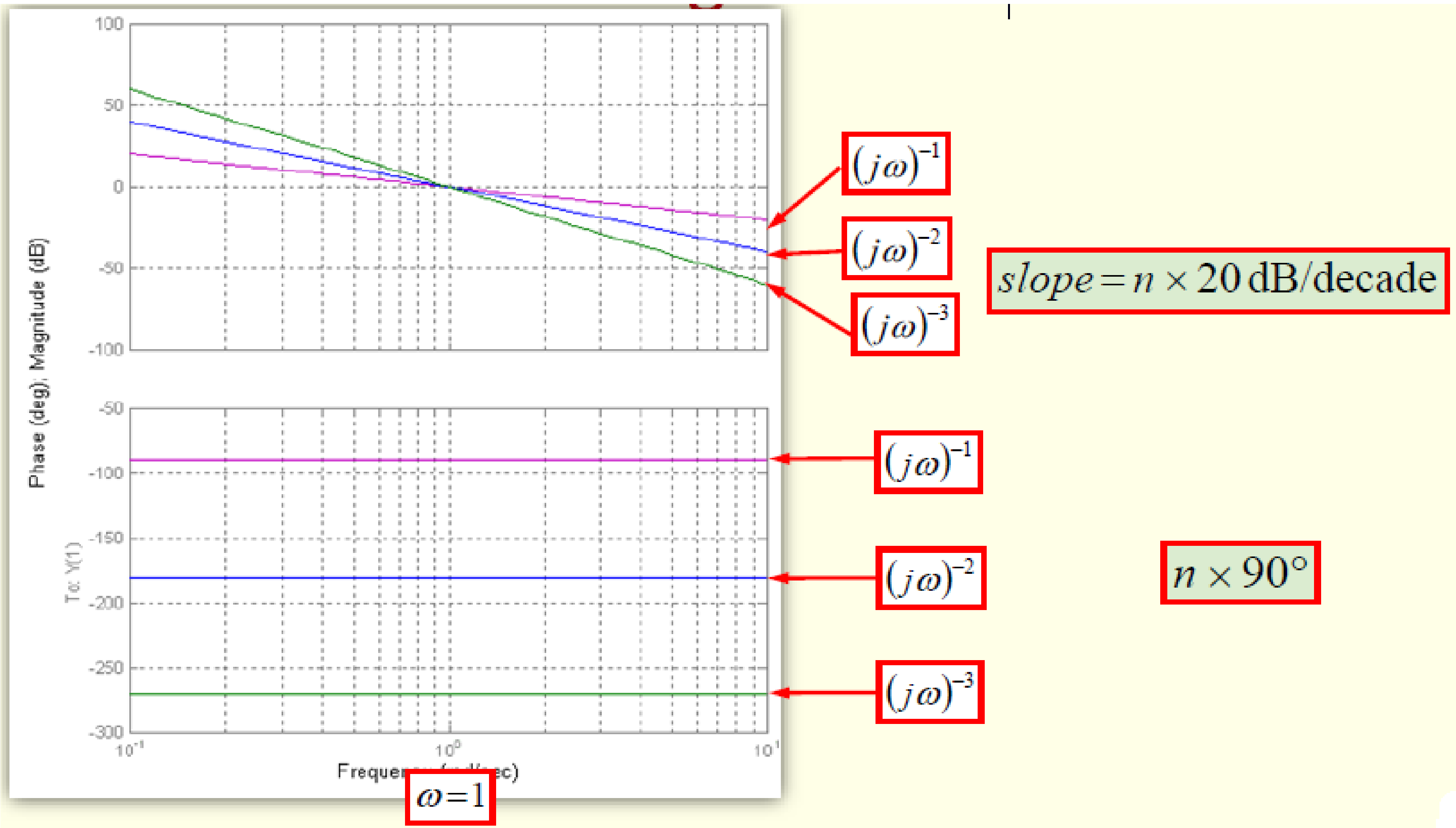
The Bode Plot for a constant gain K



The Bode Plot for a derivative factor



The Bode Plot for an integral factor



The Bode Plot for a first order factor $(1+\frac{s}{a})$

Magnitude:

$$\left|(1+j\frac{\omega}{a})\right|_{dB} = 20\log\sqrt{1+(\frac{\omega}{a})^2}$$

$$=10\log[1+(\frac{\omega}{a})^2]$$

$$\omega \prec\prec a \Rightarrow \frac{\omega}{a}\approx 0 \Rightarrow dB = 10\log 1 = 0$$

$$\omega \succ\succ a \Rightarrow 1+j\frac{\omega}{a}\approx\frac{\omega}{a}\Rightarrow dB \approx 20\log\frac{\omega}{a}$$

$$dB = 20\log\omega - 20\log a$$

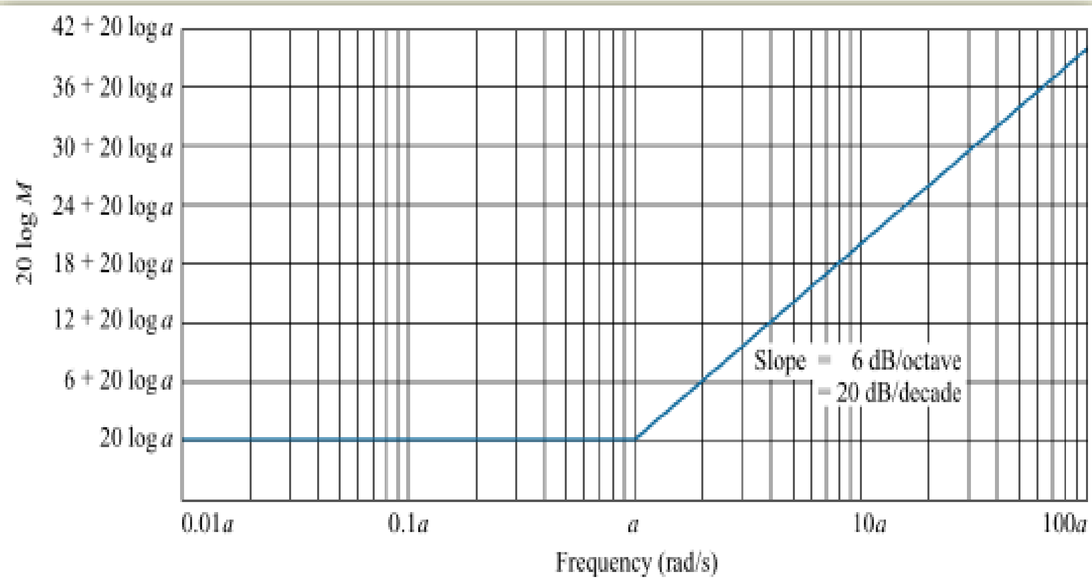
$$\omega = a \Rightarrow 1+j1 \Rightarrow dB = 10\log 2 = 3.01$$

Phase:

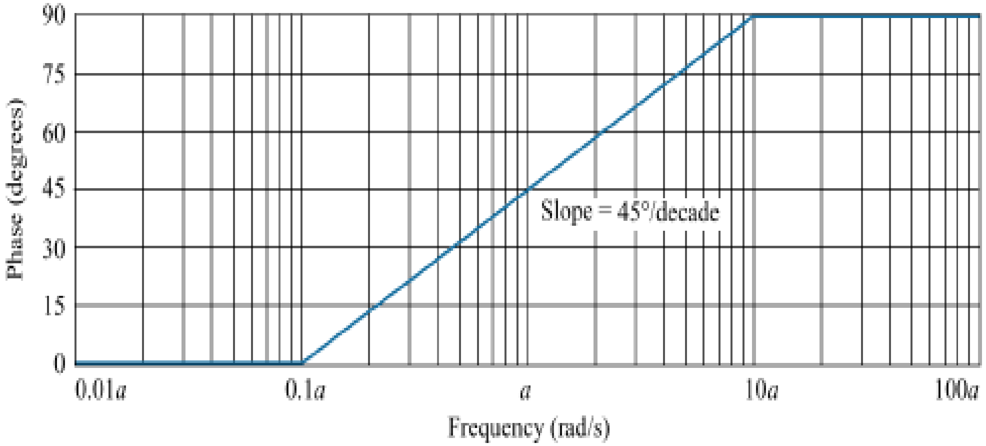
$$\angle(1+j\frac{\omega}{a}) = \tan^{-1}\frac{\omega}{a}$$

$$\omega \prec\prec a \Rightarrow \frac{\omega}{a}\approx 0 \Rightarrow \angle GH \approx \tan^{-1}0 = 0^o$$

$$\omega \succ\succ a \Rightarrow \frac{\omega}{a}\approx\infty \Rightarrow \angle GH \approx \tan^{-1}\infty = 90^o$$



(a)



(b)

The Bode Plot for a Second order factor

$$G(s)H(s)=\left(1+\frac{2\zeta}{\omega_n}s+\left(\frac{s}{\omega_n}\right)^2\right)$$

$$G(j\omega)H(j\omega)=\left(1+\frac{2\zeta}{\omega_n}j\omega+\left(\frac{j\omega}{\omega_n}\right)^2\right)$$

$$G(j\omega)H(j\omega)=(1-(\frac{\omega}{\omega_n})^2)+j2\xi\frac{\omega}{\omega_n}$$

$$\left|G(j\omega)H(j\omega)\right|_{dB}=20\log_{10}\sqrt{\left(1-(\frac{\omega}{\omega_n})^2\right)^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}$$

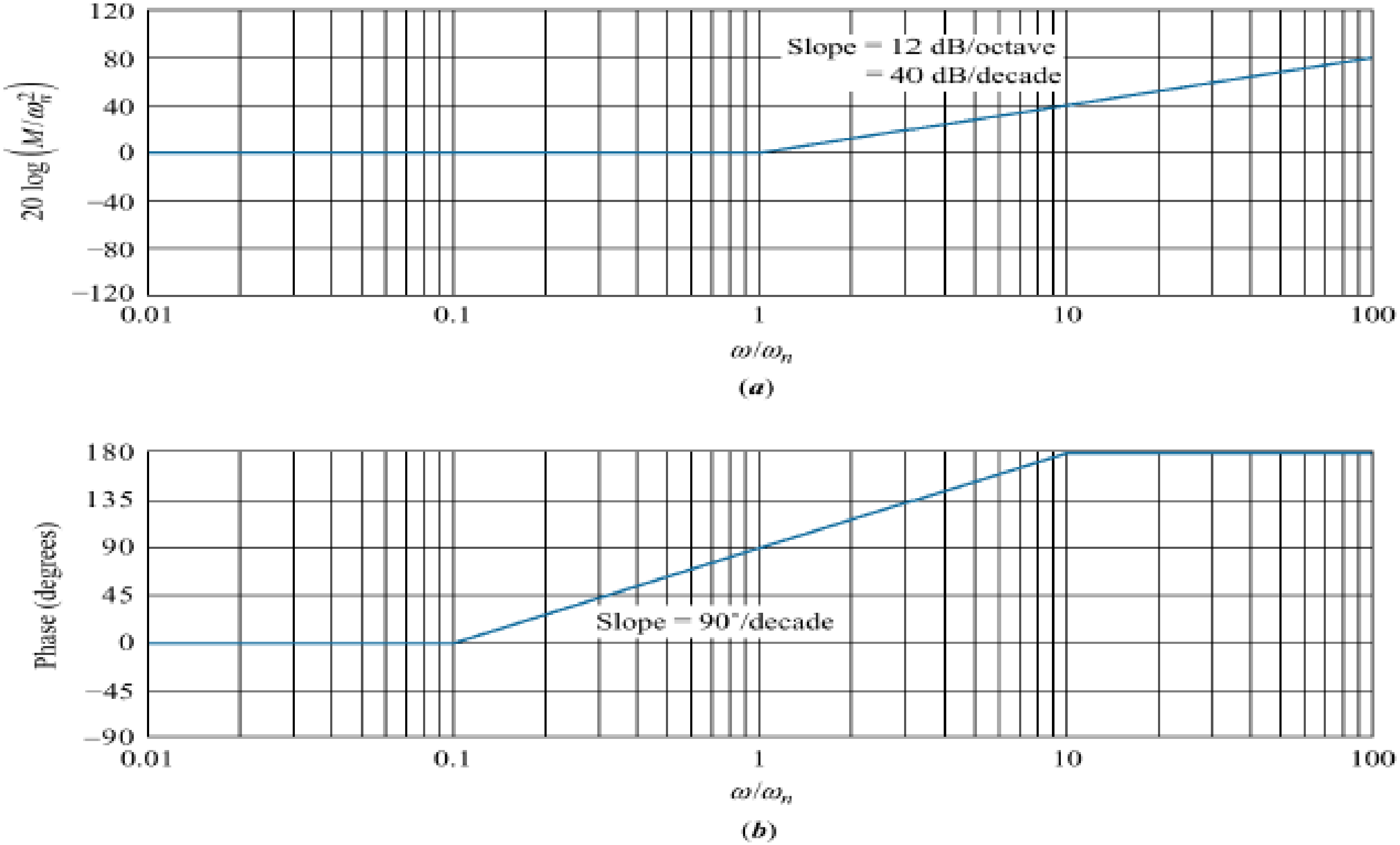
$$G(j\omega)H(j\omega)=(1-(\frac{\omega}{\omega_n})^2)+j2\xi\frac{\omega}{\omega_n}$$

$$\angle G(j\omega)H(j\omega)=\tan^{-1}\frac{2\xi\frac{\omega}{\omega_n}}{1-(\frac{\omega}{\omega_n})^2}$$

$$\left|G(j\omega)H(j\omega)\right|=\begin{cases}0 & ,\frac{\omega}{\omega_n}\prec\prec 1\\ 20\log(2\xi) & ,\frac{\omega}{\omega_n}=1\\ 40\log(\frac{\omega}{\omega_n}) & ,\frac{\omega}{\omega_n}\succ\succ 1\end{cases}$$

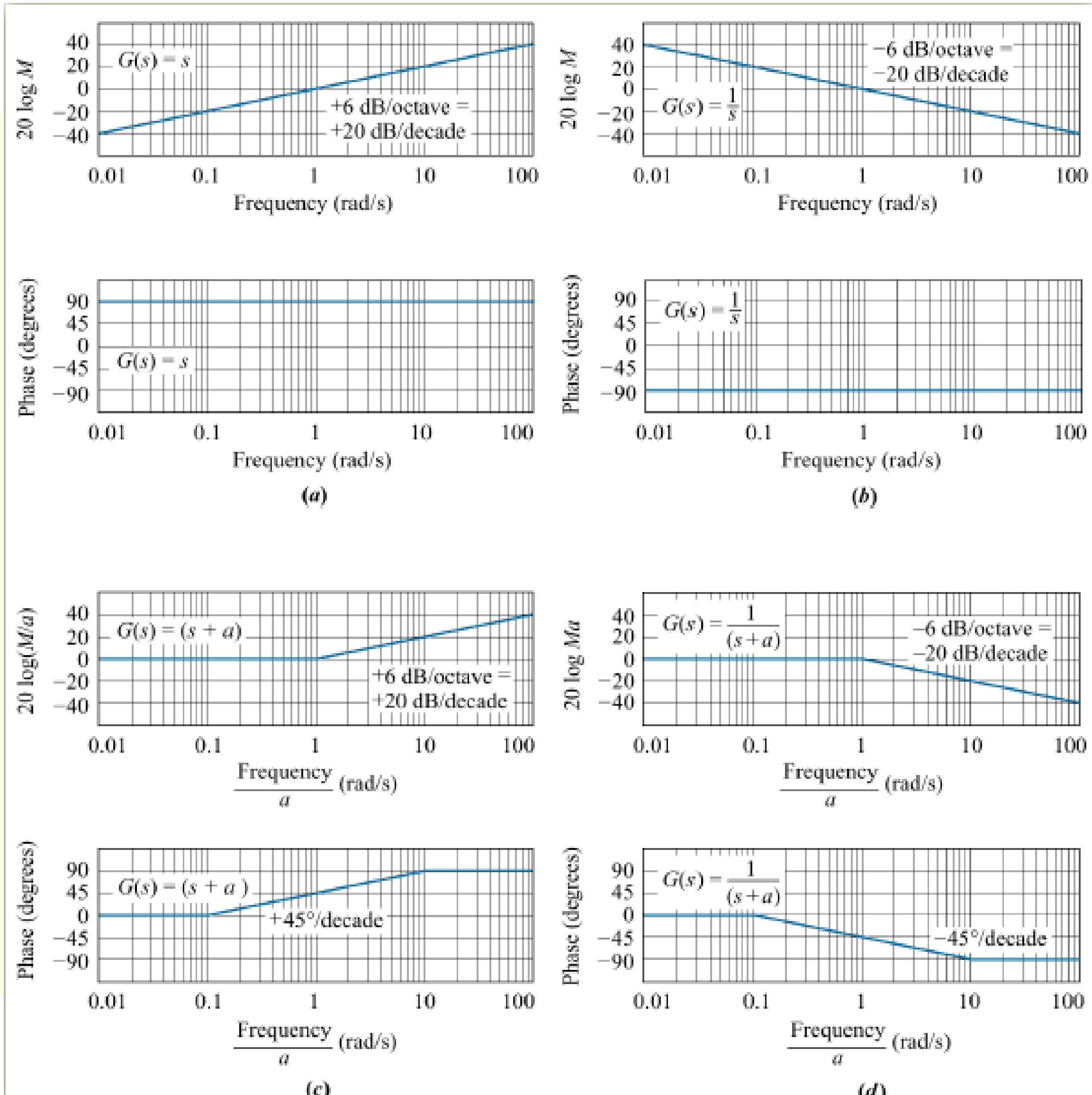
$$\angle G(j\omega)H(j\omega)=\begin{cases}0^0 & ,\frac{\omega}{\omega_n}\prec\prec 1\\ 90^0 & ,\frac{\omega}{\omega_n}=1\\ 180^o & ,\frac{\omega}{\omega_n}\succ\succ 1\end{cases}$$

$$G(j\omega)H(j\omega) = \left(1 + \frac{2\zeta}{\omega_n} j\omega + \left(\frac{j\omega}{\omega_n}\right)^2\right)$$



Bode plots for

- $G(s)H(s) = s;$
- $G(s)H(s) = \frac{1}{s};$
- $G(s)H(s) = \left(1 + \frac{s}{a}\right)$
- $G(s)H(s) = \frac{1}{\left(1 + \frac{s}{a}\right)}$



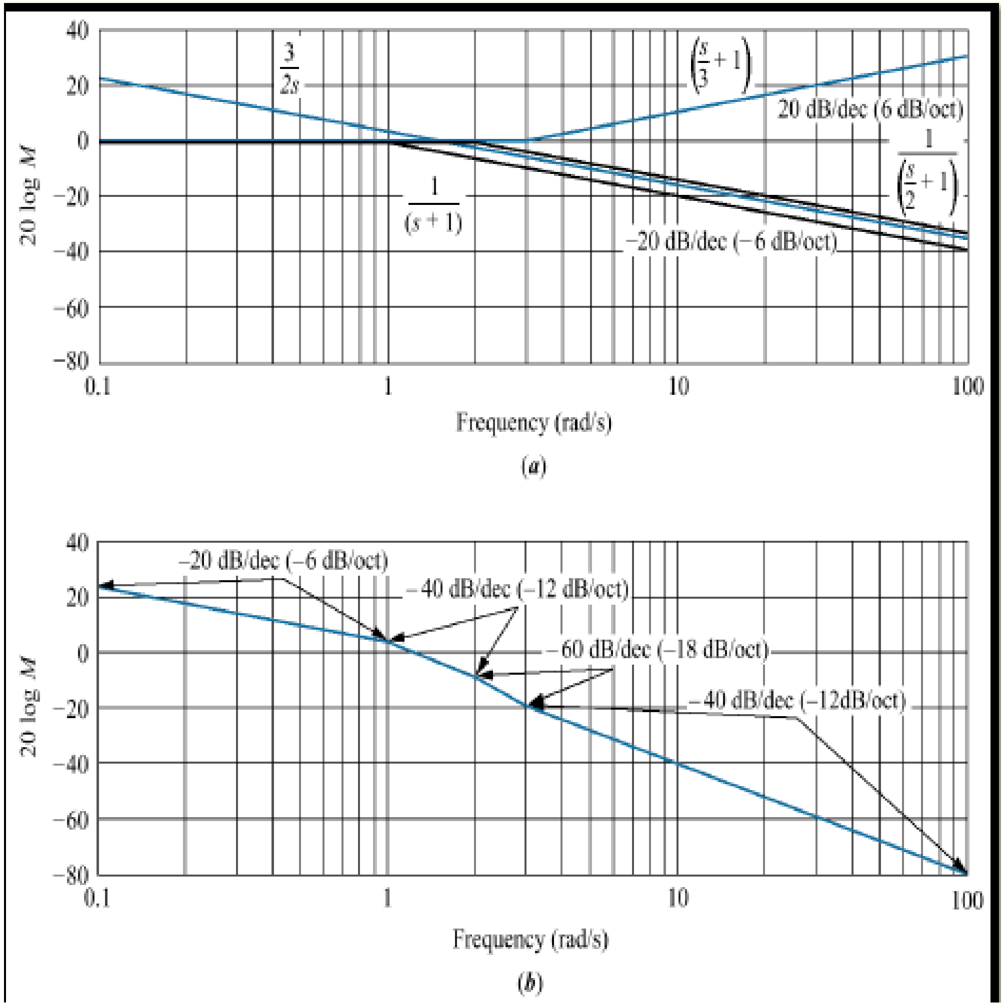
Example

- Draw the Bode plot for a system transfer function T(s)

$$G(s)H(s) = \frac{(s+3)}{s(s+1)(s+2)}$$

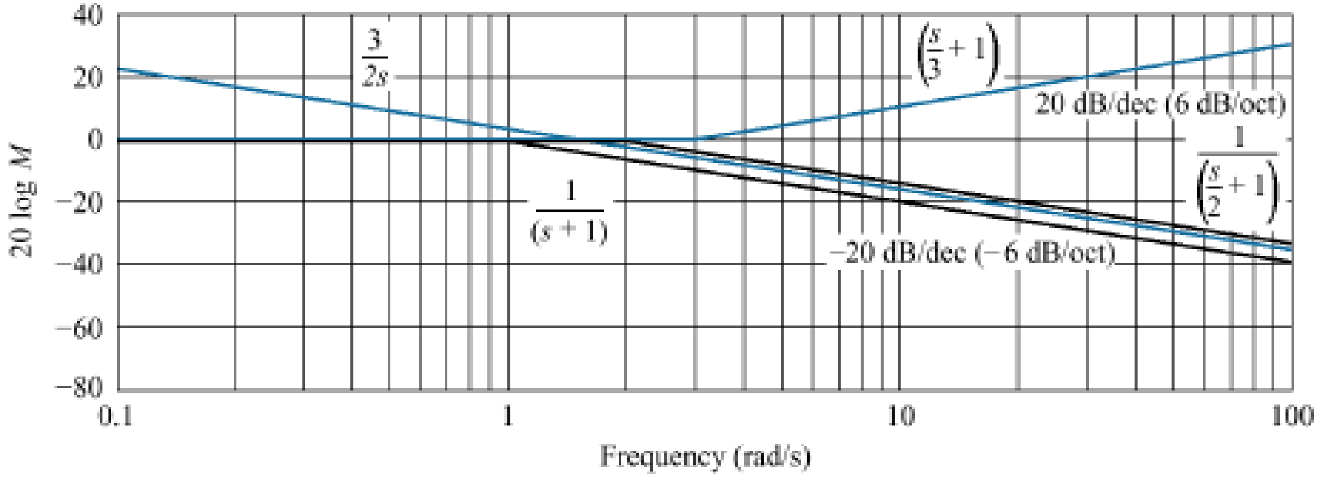
$$G(s)H(s) = \frac{3\left(\frac{s}{3}+1\right)}{2s(s+1)\left(\frac{s}{2}+1\right)}$$

$$G(s)H(s) = \frac{1.5\left(\frac{s}{3}+1\right)}{s(s+1)\left(\frac{s}{2}+1\right)}$$



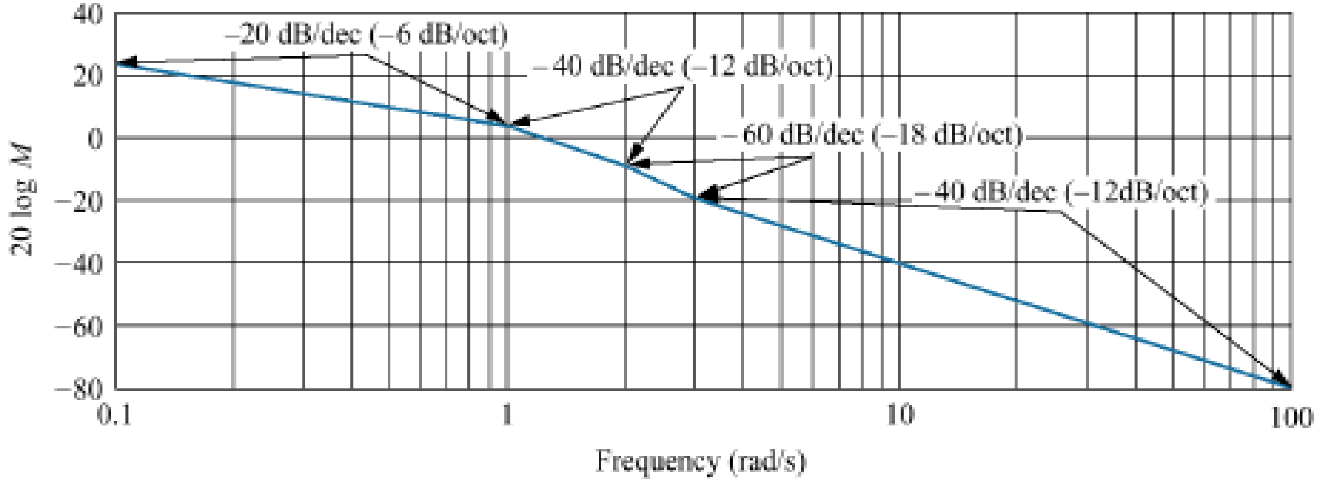
Bode Magnitude plot:
a. components;
b. composite

solution



40
20
0
−20
−40
−60
−80
20 log M
3/2s
(s/3 + 1)
20 dB/dec (6 dB/oct)
1/(s/2 + 1)
1/(s + 1)
−20 dB/dec (−6 dB/oct)
0.1
1
10
100
Frequency (rad/s)

(a)



40
20
0
−20
−40
−60
−80
20 log M
−20 dB/dec (−6 dB/oct)
−40 dB/dec (−12 dB/oct)
−60 dB/dec (−18 dB/oct)
−40 dB/dec (−12dB/oct)
0.1
1
10
100
Frequency (rad/s)

(b)

With Our Best Wishes

Automatic Control (2)

Course Staff

Thank You
For Your Attention

Mohamed Ahmed Ebrahim

Associate Prof. Dr . Mohamed Ahmed Ebrahim